Give a geometric description

$x^2+y^2=r^2$

Find the distance between P_1 and P_2

$\overline{p_1p_2}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Find the center and radius of the sphere

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=a^2$

1. Group terms 2. Complete square
3. Binomial form 4. apply formula

Find the component form and magnitude of the vector.

component form $K<v_1,v_2>=\langle Kv_1,Kv_2\rangle$

Magnitude $|\vec v|=\sqrt{V_1^2+V_2^2+V_3^2}$ length

direction $\vec v=\frac{\vec v}{|\vec v|}$

find the component form and magnitude of the $\vec u+\vec v$. $\vec u+\vec v=<u_1+v_1,u_2+v_2>$

Find the component form of the vector $\overrightarrow{P_1P_2}$ where $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$.

$\overrightarrow{P_1P_2}=<x_2-x_1,y_2-y_1>$

Vector form

$\overrightarrow{P_1P_2}=(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k$

midpoint $=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2},\frac{z_1+z_2}{2}\right)$

Dot product $\vec u\cdot\vec v=u_1\cdot v_1+u_2\cdot v_2$

Multiply $\sqrt{}$ $=3\sqrt2\cdot\sqrt{10}=3\sqrt{2\cdot10}=3\sqrt{20}$

cosine of angle between vectors $cos\theta=\frac{u\cdot v}{|u|\cdot|v|}$ $proj_v u=\left(\frac{u\cdot v}{|v|^2}\right)v$

scalar component $\vec u\to\vec v=\frac{u\cdot v}{|v|}$ angle between vectors $\theta=\cos^{-1}\frac{u\cdot v}{|u|\cdot|v|}$ RADIANS

find equation for $\vec v=ai+bj$ perpendicular to line $ax+by=c$.

1. substitute $\vec v$ in $ax+by=c$
2. substitute P for x and y to find c
3. to graph line, set $x=0$ for y-intercept; set $y=0$ for x-intercept

find equation for $\vec v=ai+bj$ parallel to line $bx-ay=c$.

1. substitute $\vec v$ in $bx-ay=c$.

Work $W=\vec F\cdot\vec D$ $\vec D=xi+yj$ from P.

or $W=(|F|\cos\theta)|D|$

$\theta=$ < 90° acute
$\theta=$ 90° right
$\theta=$ > 90° obtuse

Find the acute θ between lines.

1. determine $\vec v$ parallel to line 1 and 2
2. Find $\vec u\cdot\vec v$
3. Find magnitudes of $\vec u$ and $\vec v$
4. Find $\theta=\cos^{-1}$ ans. in radians

Cross Product $\vec u\times\vec v=\begin{vmatrix}i&j&k\\u_1&u_2&u_3\\v_1&v_2&v_3\end{vmatrix}$ $\begin{vmatrix}a&b\\c&d\end{vmatrix}=ad-bc$

$\vec u\times\vec v=\begin{vmatrix}u_2&u_3\\v_2&v_3\end{vmatrix}i-\begin{vmatrix}u_1&u_3\\v_1&v_3\end{vmatrix}j+\begin{vmatrix}u_1&u_2\\v_1&v_2\end{vmatrix}k$

Find length and direction of $\vec u\times\vec v$ and $\vec v\times\vec u$.

1. $\vec u\times\vec v$
2. $|\vec u\times\vec v|$ and $|\vec v\times\vec u|$
3. $\frac{\vec u\times\vec v}{|\vec u\times\vec v|}$ and $-$[direction of $\vec u\times\vec v$]

Find the area of a triangle determined by points P, Q and R.

Area $=\frac12|\overrightarrow{PQ}\times\overrightarrow{PR}|$

1. Find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$
2. find $\overrightarrow{PQ}\times\overrightarrow{PR}$
3. find area $\frac12|\overrightarrow{PQ}\times\overrightarrow{PR}|$
4. $\frac{\overrightarrow{PQ}\times\overrightarrow{PR}}{|\overrightarrow{PQ}\times\overrightarrow{PR}|}$

$\frac{d}{dx}(\sin x)=cosx$

$\frac{d}{dx}(\cos x)=-\sin x$

$\frac{d}{dx}(\tan)=\sec^2 x$

$\frac{d}{dx}(\ln|x|)=\frac1x$

Find the volume of the parallelepiped (box) determined by $\vec u,\vec v$ and $\vec w$.

triple scalar $|u\times v|\cdot w=\begin{vmatrix}\vec u_1&\vec u_2&\vec u_3\\\vec v_1&\vec v_2&\vec v_3\\\vec w_1&\vec w_2&\vec w_3\end{vmatrix}$

$|u\times v|\cdot w=w_1\begin{vmatrix}u_2&u_3\\v_2&v_3\end{vmatrix}-w_2\begin{vmatrix}u_1&u_3\\v_1&v_3\end{vmatrix}+w_3\begin{vmatrix}u_1&u_2\\v_1&v_2\end{vmatrix}$

volume $=|(u\times v)\cdot w|$

ans. must be abs. value $|x|$ units cubed

Orthogonal vectors or perpendicular vectors if and only if $\vec u\cdot\vec v=0$

Parallel vectors if and only if $\vec u\times\vec v=0$

Find the area of the parallelogram with vertices A, B, C and D.

1. $\overrightarrow{AB}$ and $\overrightarrow{AC}$ 2. $\overrightarrow{AB}\times\overrightarrow{AC}$ ans. in square units.

Find the area of the triangle with vertices P, Q and R.

1. find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$
2. find $\overrightarrow{PQ}\times\overrightarrow{PR}$
3. find area $\frac12|\overrightarrow{PQ}\times\overrightarrow{PR}|$ ans. in square units.

A line passes through a point P and its parallel to a vector $i+j+k$.

Parameterize $v_1i+v_2j+v_3k$ (x_0,y_0,z_0)

$x=x_0+tv_1$ $y=y_0+tv_2$ $z=z_0+tv_3$

for $-\infty<t<\infty$

A line passes throug points P and Q. find stadard parametric equations.

1. find $\overrightarrow{PQ}$ 2. Parameterize

A line passes through a point P and its perpendicular to the plane $Ax+By+Cz=D$

1. Convert $Ax+By+Cz=D$ to normal vector $i+j+k$
2. Parameterize

Find a parametrization for the line segment joining the points P and Q.

1. Find $\overrightarrow{PQ}$
2. Parameterize
3. what value of t will result in point P and point Q.

Find the equation for the plane through the points P, Q and R.

1. find $\overrightarrow{PQ}$ and $\overrightarrow{PR}$
2. find $\overrightarrow{PQ}\times\overrightarrow{PR}$
3. substitute point P in $\overrightarrow{PQ}\times\overrightarrow{PR}$ in the form $Ax+By+Cz=D$

Find the equation the plane through point P perpendicular to the parametized line. $-4+at$ $P=-4$

1. Find normal vector $i+j+k$
2. Plug $i+j+k$ in A,B,C and P in (x_0,y_0,z_0)
3. $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$
4. Solve for D.

Find the distance from the point P to the line.

1. find $\overrightarrow{PQ}=i+j+k$ 2. Find $\overrightarrow{PQ}\times\vec v$
3. $|\overrightarrow{PQ}\times\vec v|$ and $|\vec v|$ 4. Distance $d=\frac{|PQ\times v|}{|v|}$

Find the distance d, from the point S. and the plane $x+y+z=0$

1. plane to vector form $i+j+k$. $\vec v$
2. to find point P use x-int let $y=0$ $z=0$
3. Find $\overrightarrow{PS}$ then $d=\left|\frac{ps\cdot v}{|v|}\right|$

Find the angle between the planes $x+y+z=0$ and $x_1+y_1+z_1=0$

1. Find normal vectors $i+j+k$
2. find angle $\theta=\cos^{-1}\frac{u\cdot v}{|u|\cdot|v|}$ RADIANS

Find the point P, at which the line intersects the plane.

1. Substitute each coordinate in the equation of the plane.
2. Simplify by multiplying each polynomial.
3. Solve for t. Substitute back into the parameterization line.

Find a parametrization of the line for which planes intersect.

1. Find normal vectors $\vec n_1$ and $\vec n_2$.
2. Find $\vec n_1\times\vec n_2=\vec n$.
3. substitute $z=0$ into the plane equations and solve simultaneously for x and y.
4. Give x,y,z coordinates. Then, parameterize parallel to $\vec n$.

Find the limit 1. find the limit of each component 2. Evaluate limits.

the position of a particle in the xy-plane at time t is $r(t)=(t+3)i+(t^2-4)j$. Find an equation in x and y whose graph is the path of the particle. Then find the particle's velocity and acceleration vectors at $t=c$.

1. x-coord. equal to i. 2. solve for t in terms of x. $(x+a)^2=x^2+2ax+a^2$
3. substitute x in y and solve for t. 5. Find velocity vector at $t=c$. by plugging t.
4. find velocity $v(t)=\frac{dr}{dt}=\frac{df}{dt}i+\frac{dg}{dt}j$
6. Find the acceleration vector a(t), take derivative of v(t). No step 5.

the path $r(t)=(3\cos t)i+(3\sin t)j$ describes the motion on the circle $x^2+y^2=9$. Find the particle's velocity and acceleration vectors at $t=\frac{\pi}{2}$ and $t=\frac{\pi}{4}$. 1. find velocity $v(t)=\frac{dr}{dt}=\frac{df}{dt}i+\frac{dg}{dt}j$

2. find a(t). 3. find v(t), a(t) and r(t) position at $t=\frac{\pi}{2}$
4. repeat for $t=\frac{\pi}{4}$.

The equation $r(t)=(3t+6)i+(7t^2-6)j+(4t)k$ is the position of a particle in space at time t. 3. Find direction $=\frac{\vec v}{|\vec v|}$

1. v(t) and a(t) 2. Find speed $=|v(t)|$ + speed at t.
4. $v\cdot\left(\frac{\vec v}{|\vec v|}\right)$

The equation $r(t)=(t+2)i+\left(\sqrt3 t\right)j+\left(2t^2\right)k$ is the position of a particle in space at time t. Find the angle between the velocity and acceleration at time $t=0$. 3. find angle $\theta=\cos^{-1}\frac{u\cdot v}{|u|\cdot|v|}$

1. v(t) and a(t) 2. Find a(t) at t.

find parametric equations for the line that is tangent to the given curve at the given parameter value $r(t)=(3\sin t)i+(t^2-\cos t)j+(4e^t)k$, $t=0$.

1. Find r(t) at t=0 to get tan line (x,y,z).
2. Find velocity $v(t)=\frac{dr}{dt}=\frac{df}{dt}i=\frac{dg}{dt}j$ of r(t). 3. evaluate v(t) at t=0.
4. parametize tan line (x,y,z) parallel to v(0)

Equation r(t), $t\geq0$ describes the motion of a particle moving along the unit circle.

1. v(t) and a(t) then speed 2. v·a

evaluate integral 1. if applicable, use u substitution.
3. evaluate limits of integration.

$t=0\to u=0$ $u=6t^2=6\cdot0^2=0$ adjust integral boundaries
$t=1\to u=6$ $u=6t^2=6\cdot1^2=6$

solve the initial value problem for $\vec r$ as a vector function of t.

1. integrate $\vec r$. 2. evaluate $\vec v(0)=\vec r(t)$ to find C.
3. substitute C in $\vec r(t)$.

solve the initial value problem for $\vec r$ as a function of t. $\frac{d^2r}{dt^2}=$

1. $\int r(t)\,dt=R(t)+C$ 2. use $\frac{dr}{dt}|_{t=0}$ to find constants at t=0.
3. $\int\frac{dr}{dt}+C$ 4. find values of C by using r(0).

At time t=0, a particle is located at the point (8,8,8).
It travels in a straight line to the point (2,2,7).

1. $\int v(t)$ constant accl. 2. Find $\overrightarrow{PS}$ 3. find $|v|\cdot\left(\frac{\vec v}{|\vec v|}\right)$ = velocity
4. find v(t) at t=0. 5. plug v(t) t=0 in velocity. 6. like terms
7. integrate $\int dr$ 8. find r(t) at t=0 9. $C_2=r(0)$ (8,8,8)
10. substitute in r(t) 11. solve.

A projectile $R=(v_0\cos\theta)t$ multip. m/sec by 1000 0.000 km/sec

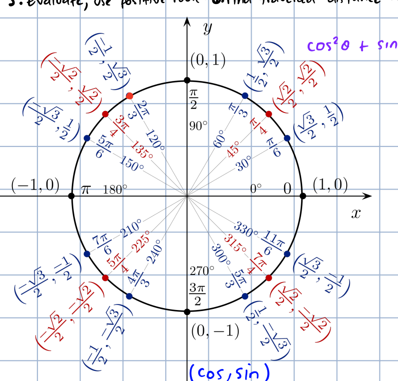
$v_0=860\,\frac{m}{sec}$ initial speed $\theta=60°$ angle of projection $R=21$ km

An athlete puts a 16-lb shot at an angle of 60° P(x,y)

$r=(x_0+(v_0\cos\alpha)t)i+\left(y_0+(v_0\sin\alpha)t-\frac12gt^2\right)j$

v_0 = initial speed
α = launch angle
g = gravity at 32.

1. Find travel time 2. rearrange terms for y formula
3. $=\left(-\frac{g}{2}\right)t^2+(v_0\sin\alpha)t+y_0=At^2+Bt+C$ $t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
4. Find t when y=0, use quadratic formula. ans. in feet
5. Evaluate, use positive root. 6. Find travelled distance 7. use x formula with t value.



(cos, sin)

$\cos^2\theta+\sin^2\theta=1$ $\int e^u\,du=e^u$

$(e^u)=\frac1e$

$\int x^a\,dx=\frac{x^{a+1}}{a+1}$

$e^0=1$

$d/dx\,(x^n)=nx^{n-1}$

$d/dx\,(e^x)=e^x$

$\int\frac1u\,du=\ln|u|$

$\frac{d}{dt}e^{at}=ae^{at}$

Chain rule $F'(g(x))=F'(g(x))\,g'(x)$

$e^{\ln x}=x$

$\int a\,dx=ax$

$\sqrt[n]{a}=a^{\frac1n}$

$\frac{x^a}{x^b}=x^{a-b}$

$\frac{a}{2}\cdot\frac{2}{3}u^{\frac32}$ $\frac{1}{3}u^{\frac32}\cdot\frac{3}{2}$

FORMULAS

Degree	Radians	$sin\ \theta$	$cos\ \theta$	tan θ	$cosec\ \theta$	$sec\ \theta$	$cot\ \theta$
0°	0	0	1	0	-	1	-
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{2}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{2}$	2	$\frac{\sqrt{3}}{3}$
90°	$\frac{\pi}{2}$	1	0	-	1	-	0
120°	$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	-2	$-\frac{\sqrt{3}}{3}$
135°	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$\sqrt{2}$	$-\sqrt{2}$	-1
150°	$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	2	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{3}$
180°	π	0	-1	0	-	-1	-
270°	$\frac{3\pi}{2}$	-1	0	-	-1	-	0
360°	2π	0	1	0	-	1	-

COMMON DERIVATIVES

$$\frac{d}{dx}(x) = 1$$
$$\frac{d}{dx}(\sin x) = \cos x$$
$$\frac{d}{dx}(\cos x) = -\sin x$$
$$\frac{d}{dx}(\tan x) = \sec^2 x$$
$$\frac{d}{dx}(\sec x) = \sec x \tan x$$
$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$
$$\frac{d}{dx}(\cot x) = -\csc^2 x$$
$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}(a^x) = a^x \ln(a)$$
$$\frac{d}{dx}(e^x) = e^x$$
$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$$
$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$
$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$$

COMMON FACTORING EXAMPLES

$$x^2 - a^2 = (x+a)(x-a)$$
$$x^2 + 2ax + a^2 = (x+a)^2$$
$$x^2 - 2ax + a^2 = (x-a)^2$$
$$x^2 + (a+b)x + ab = (x+a)(x+b)$$
$$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$$
$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$
$$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$$
$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$